FROM FLATLAND TO FRACTALAND: NEW GEOMETRIES IN RELATIONSHIP TO ARTISTIC AND SCIENTIFIC REVOLUTIONS

RHONDA ROLAND SHEARER
62 Greene Street, New York, NY 10012
Fax: (212)925-0459 Internet: 74577.676@compuserve.com

Abstract

Abbott's 19th century book, *Flatland*, continues to be popularly interpreted as both a social commentary and a way of visualizing the 4th-dimension by analogy. I attempt here to integrate these two seemingly disparate readings. *Flatland* is better interpreted as a story with a central theme that social, perceptual, and conceptual innovations are linked to changes in geometry.

In such cases as the shift from the two-dimensional world of Flatland to a three-dimensional Spaceland, the taxonomic restructuring of human importance from Linnaeaus to Darwin, or the part/whole proportional shift from Ptolemy's earth as the center of the universe to Copernicus's sun, new geometries have changed our thinking, seeing, and social values, and lie at the heart of innovations in both art and science. For example, the two greatest innovations in art — the Renaissance with geometric perspective, and the birth of modern art at the beginning of this century with n-dimensional and non-Euclidean geometries — were developed by artists who were thinking within new geometries.

When we view the history of scientific revolutions as new geometries, rather than only as new ideas, we gain direct access to potential manipulations of the structures of human innovation itself. I will discuss the seven historical markers of scientific revolutions (suggested by Kuhn, Cohen, and Popper), and how these seven traits correlate and can now be seen within the new paradigm of fractals and nonlinear sciences.

Edwin A. Abbott's 1884 book, *Flatland*, has continued to be, as one reviewer then wrote, "an enigma.[1]" One has to ask, after reading *Flatland,* what does visualizing the 4th-dimension have to do with a satire of Victorian culture? Both "the 4th-dimensional analogy" and "social commentary" have consistently been the dual, yet seemingly incompatible, interpretations of *Flatland* since its inception. Geometry, surely the most Platonic of disciplines (conventionally viewed as transcending culture), *is* indeed oddly paired with social satire in the *Flatland* tale.

The first fifty pages of this hundred-page book discusses how Flatlanders live in their two-dimensional world and is the primary source for interpreting *Flatland* as a social commentary. As told by A Square, a middle-class lawyer and main character (also the eventual hero and martyr of *Flatland*), the book's first half discusses Flatland's history and social rules (low class males are triangles, middle class are squares, high class are multi-sided polygons, and circles are priests; women of all classes are lines). In Flatland, males and females have two separate languages based on gender (males speak in an educated language of "science" and are only patronizing females when they speak to them in the exclusively female language of "feelings").

The second half of the book leads to the interpretation of *Flatland* "as a way to visualize the 4th-dimension by analogy." Here, A Square is "visited" by A Sphere from our three-dimensional world (called "Spaceland" by A Sphere). A Sphere's mission is to appear to an average Flatland citizen at the beginning of every millennium. His new appearance, his third, was timed for December 31, 1999 — for the purpose of revealing, to an unsuspecting Flatlander, the truth about the existence of worlds beyond their two-dimensions.

Interestingly, later in the story, it is A Square, not his mentor A Sphere, who, after visiting Sphereland, Pointland, and Lineland, has a great insight, through deduction, of the importance of "analogy" itself. A Square concludes by reason, based on his experiences of 0, 1, 2, and 3 dimensions, that with "surety" a 4th, 5th, 6th, and 7th-dimension and beyond must exist. The theoretical necessity of higher or unending dimensions can be understood by logic and analogy, even though we have neither seen higher dimensions nor know them directly in any way.

Thus, the cloud frontispiece of *Flatland* (See Fig. 1) with its emphasis on numerous dimensions (ten instead of the 0–3 explored in the story), records Abbott's message — as embodied in A Square's repeatedly stated and passionate desire, that, "by any means, I want to arouse in the interiors of Plane and Solid Humanity a spirit of rebellion against the Conceit which would limit our Dimensions to Two or Three or any number short of Infinity."

As stated earlier, a feeling of contrast between the first and second halves of the book (social commentary in the first, and A Square's experiences of the 3rd, lst, and 0 dimensions in the second) represents the traditional interpretation of two disconnected parts. After reading Abbott's biographical history, old reviews, and surveying various scholarly interpretations and introductions to the many editions of *Flatland* since 1884, I developed a new interpretation that, I believe, not only integrates the first and second halves of the book (the social commentary and the 4th-dimensional analogy), but is also relevant to the arrival of fractal geometry into our present culture.

My new conclusions about Abbott's book helped to confirm and to give a "Flatland metaphor" for concepts I had previously been considering, and that I now call the Flatland Hypothesis. In summary, the Flatland Hypothesis[2] holds that: (1) similar geometries underlie perception and cognition, as well as social and physical organization; (2) therefore,

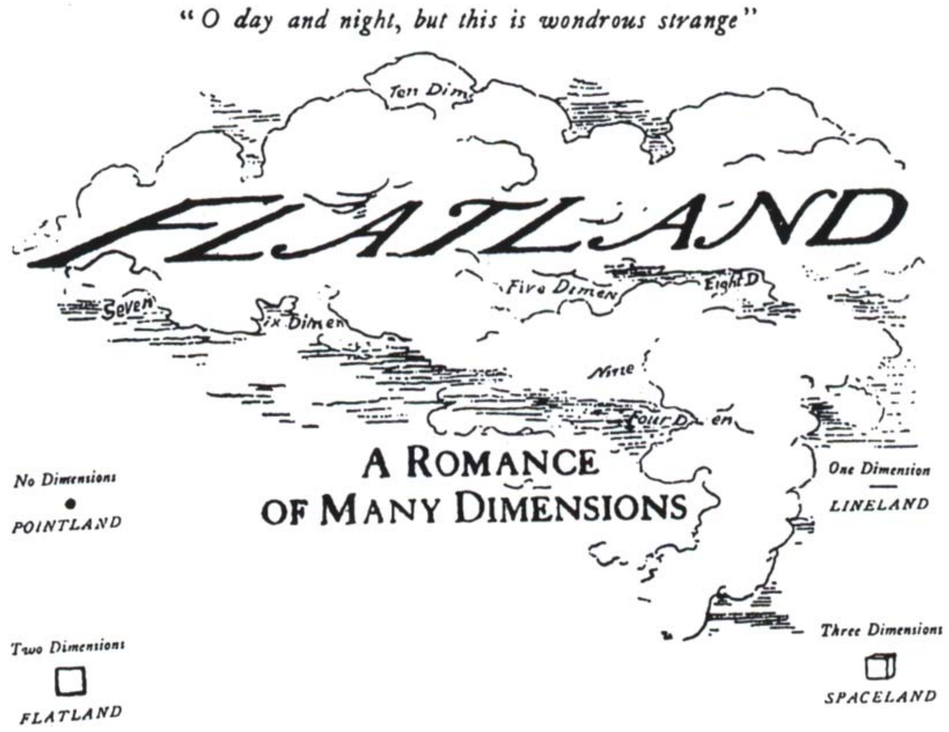

Fig. 1 Cloud frontispiece from *Flatland* illustrates the emphasis Abbott places on the importance of "new geometries" as being connected to unending progress and human change or innovation.

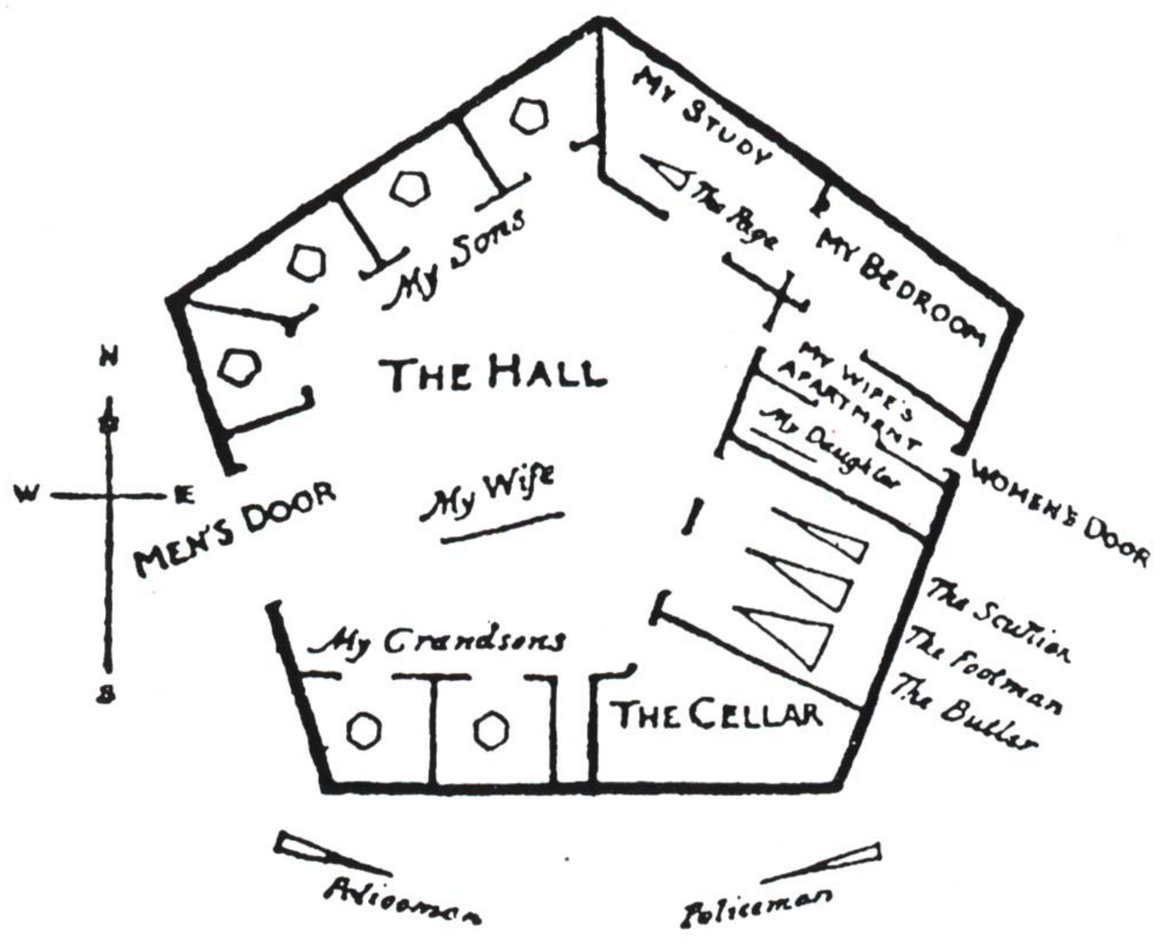

Fig. 2 A Square's pentagon house is only one of Abbott's geometric descriptions of Flatland. Everything about Flatland was geometric, how they lived (in pentagons), how they looked (squares, circles, lines, etc.), what they believed ("configuration makes the man"), even how they recreated (thinking about geometry).

changes in these geometries are essential in altered thinking, seeing, and social values that, in large scale cases, we experience as innovation, creativity, or paradigm shift.

As a fundamental step toward my new interpretation of Flatland, I chose an opposite approach to the standard interpretation of Flatland as both a Victorian satire and 4th-dimensional analogy, and asked the reverse question: "instead of the differences, what are the *similarities* between the first and second half of the book?" By thought experiment, I was able to see that, by looking for the *similarities* between the first and second halves of the book, a pattern emerged.

The social satire of the book's first half is essentially a series of descriptions of Flatland's social and physical *structures* — that is, their geometries. Everything about Flatland is geometric — how they lived (in pentagons see Fig. 2), how they looked (squares, circles, lines etc.), how they recreated (thinking about geometry), what they believed ("Configuration makes the man." A Flatlander's class and value is judged solely by shape, an "unquestionable doctrine," A Square tells us). What Abbott cleverly shows (similar to the formal insight credited later to Thomas Kuhn, author of the *Structures of Scientific Revolutions*), is an apparent linkage of thinking to perception as demonstrated by individuals within their experience of conceptual change.[3]

As in the Chinese proverb, "a square bowl creates square water," the Flatlanders' social structures shape their beliefs. In the book's first half, this connection of perception, cognition, and geometry is continually repeated. For example, if a Flatlander is visually identified as an "irregular," this *perception* is simultaneously a *concept* (as when A Square informs us that "irregularity" is "equated with moral obliquity and criminality, and is treated accordingly"). In Flatland, perception (irregular shape) and concept (dualism between the "regulars" and the "irregulars" — the "regulars" hierarchical importance, and the irregular's "down" position or devaluation) are both essentially *geometries.*

With this point in mind, upon re-reading, we can see the extent of exaggeration Abbott uses to make us aware that what Flatlanders perceive and what they believe and think are linked to common structures or geometries. But most importantly, Abbott also wanted to stress that the limits of this *Flatland* reality, and its underlying and related geometric structures, should strike as familiar, as obvious analogies to the perception, cognition, and social and physical structures in our own world of experience.

By understanding the first half of the book as Abbott's underscoring of the geometric connections among seeing, thinking, and social structures, we can now understand how the second half of Flatland connects with the first — but with an added twist.

The second half shows how the *status quo* of rigid, social, conceptual, and perceptual structures can be changed through an individual's revelation, insight, and transformation arising from the acquisition of new geometries. Clearly this happened to A Square. After A Square experienced new geometries (0, 1, and 3 dimensions), he could no longer maintain his former beliefs that Flatland represents a true or absolute reality. A Square's perception, thinking, and social values were *literally* reconfigured and transformed, integrated and updated when he assimilated a new world view. A Square's direct experiences and understanding moved from 2 dimensions to knowledge of, now, 0 through 3 dimensions.

From Abbott's historical point of view, non-Euclidean and n-dimensional geometries' struck his Victorian culture with a shock similar to the blow of A Sphere arriving into Flatland. As a headmaster and clergyman interested in learning, religion and mathematics, Abbott would have understood the debate and concern about the philosophical status of Euclid's axioms, formerly accepted as absolute truth, but now challenged. This dethronement of classical geometry occurred as a result of non-Euclidean geometry's alternative to the parallel postulate (Euclid's idea that parallel lines will never meet, and that the angles of a triangle always add up to 180° though these concepts were no longer true in non-Euclidean geometry's curved space).

"Euclid's Axioms" was the most important text in 19th century Britain, next to the Bible.[4] The unassailability and self evident logic of Euclid was used by the clergy as equal to and analogous with the self evident truth of the existence of God. In response to non-Euclidean geometry, some theologians went so far as to write that non-Euclidean geometry

was "satanic." Mathew Ryan, a 19th century theologian, referred to non-Euclidean geometry as a "beastly foolish" *Invention*[5];

> The divine nature of Euclidean space,
> wherein we dwell, is *eternal, simple,*
> *continuous, homogeneous,* and *immutable.*
> "Non-Euclidean" space is the false
> invention of *demons*, who'd gladly
> furnish the dark understandings of the
> "non-Euclideans" with false knowledge.
> How foolish are the boastings of the non-
> Euclideans compared with the Logical
> Teachings of the meek followers of Jesus

Abbott was well aware not only of mathematics and religion but of the importance of scientific revolutions in conceptual change, as evidenced from his other essays and writings. He refers to, and relates to the Bible, the conceptual transformation from the "Ptolemaic to the Copernican or Newtonian thought" as a specific example.[6] I believe that Flatland was both a satire, explanation, and a tutorial intended to address Victorian ecclesiastical fears of non-Euclidean geometry's arrival into culture. Abbott argues that we should not fear "new geometry" or see it as a threat to religion because this novelty is directly connected to the individual's experience of change and revelation.

In Flatland, Abbott combines A Square's geometric mission with a religious virtue. He mentions the three millennial visits by A Square to Flatland, an overt reference to Christianity, and he uses many other religious analogies including A Sphere calling A Square "a fit apostle for the Gospel of the Three Dimensions."

In this interpretation, Abbott makes religious and scientific revelation synonymous, showing us the connection of geometries to all aspects of our lives. These geometries are not only common features of our beliefs, but are, most importantly, *limited*; we should expect that they will continually change (like Abbott's example of the evolution from Ptolemiac to Newtonian thought). Abbott emphasizes that it is only new geometries like the non-Euclidean geometries, then new — that can save us from "arrogance" and allow large scale change in an individual's, and then later, society's perception, cognition, and values.

This analogy of Flatland, describing the shock of the arrival of a "new geometry" within a society, is, I believe, not only an analogy to non-Euclidean geometry within a Platonic culture, but simultaneously a lesson about all conceptual revolutions.

Those of us working with fractals, also know the initial resistance met by Benoit Mandelbrot from the mathematical community, after his important invention in 1975. Fractal geometry's sudden arrival into our culture, teaching us that the world consists not only of circles, lines, and squares, but mostly of irregular and fractal forms, was a shock to the *status quo* and to the tradition of searching within irregular form for Platonic perfection and its related Euclidean and non-Euclidean shapes and rules.

Let us return to this point of considering scientific and artistic revolutions not just as changing images and ideas but, more specifically, as changing geometries. Immediate examples of scientific revolutions as geometric change range from the taxonomic re-structuring of human importance from Linnaeaus to Darwin, or the part/whole proportional shift from Ptolomy's earth as the center of the universe to Copernicus's sun. The two greatest innovations in art — the Renaissance with geometric perspective, and the birth of modern art

TRAITS OF SCIENTIFIC REVOLUTIONS

1. **New Ideas**
2. **New Languages and Metaphors**
3. **New Kind/Hierarchies and Categories (Taxonomies)**
4. **New Part/Whole Relationships**
5. **New Icons (Visual Symbols)**
6. **Sense of "Conversion" or Gestalt Switching (New Thinking, Seeing, Values, "Reality")**
7. **Many Revolutions Have an "Ideological Component"**
8. **History Needs to be Rewritten for Logical and Factual Congruence**

Fig. 3 8 traits of Scientific Revolutions. Sources: Kuhn, I. B. Cohen, Popper, Thagard.

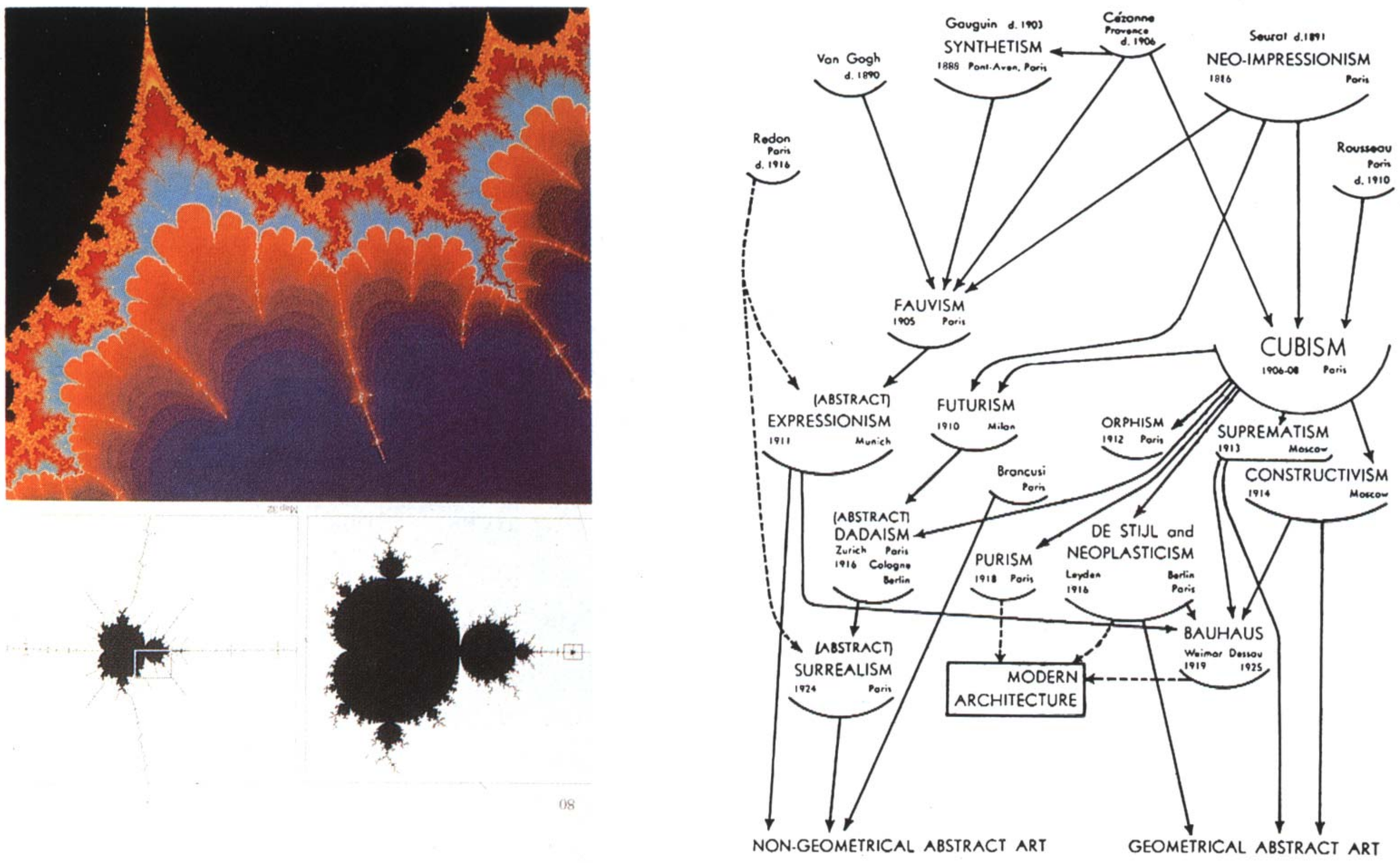

Fig. 4 Compare the Mandelbrot set by Richard F. Voss to Alfred Barr's taxonomic chart describing the "evolution" of modern art as a dichotomization of either non-geometric abstraction or geometric abstraction. With the Mandelbrot set, we can no longer categorize organic or non-geometric abstraction as separate from geometric abstraction. Fractals can be described as being equally both. The conventional categories that were once "safe" ways to identify art objects must now be rewritten.

at the beginning of this century with n-dimensional and non-Euclidean geometries were developed by artists thinking within new geometries.[7] Going from two-dimensional medieval art to three-dimensional perspective, and from three-dimensional perspective to the modern images that vary from references to higher dimensions to flat planes, both directly involve perceptual and conceptual geometric change. Renaissance artists, like Brunelleschi, were developers of perspective in a technical, mathematical sense, whereas,

modern artists explored popular interpretations of mathematicians work in n-dimensional and non-Euclidean geometries, namely, the 4th-dimension and curved space, respectively.[8]

When we view the history of scientific revolutions as new geometries, rather than as new ideas only, we are closer to the potential underlying structures of human innovation itself. Ironically, even though Kuhn's important book was entitled *The Structure of Scientific Revolutions*, it never really talked directly about structures. But by grasping the importance of new geometries, we gain direct access and can thereby focus on a new view of what's behind the conscious and nonconscious manipulation of creativity and innovation in art or science.

After reading the literature of Kuhn, I.B. Cohen, Popper, and other historians of science, I found, despite their differences, that they agree on certain characteristics or traits of scientific revolutions — (see Fig. 3).[9]

Note how many of these characteristics are already evident within the perceptual and cognitive changes created by the entrance of fractals, "a new geometry", in our culture.

Referring to Fig. 3, we now have a new language that describes the "irregular" shapes of nature (as in point #2) as well as a new part/whole relationship in our concept of the world (#4). Just as Copernicus changed our view of the universe, from the earth to the sun as prominent and at the center, Mandelbrot changed our view of the world from Platonic

Fig. 5 This Fractal Fern by Michael Barnsley can be accurately described as being literally both abstract and real.

and classical geometries to the paradigm of fractals, the irregular and nonlinear. This transformation of the regular to the irregular is a more complete fulfillment of what had already begun with Einstein's general relativity, when "regular" absolute Newtonian space became relative and curved.

Certainly, the Mandelbrot set is a powerful new icon (#5) relating visually to both computers and natural world. The taxonomies of Art History that separated the organic and geometric now have to be changed to accommodate fractals (#3 and #8), for fractals are both organic and geometric (See Figs. 4(a) and 4(b)).[10]

For me, the most startling aspect of fractal geometry lies in its relationship to characteristics of conceptual revolutions — particularly to the idea of new categories. Fractals not only defy the categories of organic and geometric, understood before as dual opposites; fractals also blur the boundaries between abstract and real. The fractal fern looks real but it is also an abstract, geometric object at the same time (See Fig. 5).

Neither category provides a more accurate description. Abstract and real are no longer dualistic or hierarchical opposites whereas, in our western tradition, abstract is considered as superior to, and more perfect than, anything real.[11]

I know of no other structures that are so completely non-hierarchical and non-dualistic. Abstract and real, reductionistic and holistic, organic and geometric, regular and irregular, any conventional dualisms or hierarchies one can specify as typical of the history of descriptions of forms and nature, now become just mental descriptions rather than judgments of relative worth, when dealing with fractal geometry. Fractals are not tailor-made for a patriarchal culture whose political agenda, social rules and beliefs (as in Flatland) are reflected in the formerly exclusive rule of classical geometries.

Following the general principle of Abbott's hope, we have now moved from Flatland to Fractaland, to a world we now recognize as being mostly fractal. Those of us pursuing this "new geometry" are the soldiers of Abbott's revolution — a "race of rebels who shall refuse to be confined to limited Dimensionality."

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10. A. H. Barr, Jr., *Cubism and Abstract Art,* 2 March–19 April 1936, The Museum of Modern Art, New York.
11. In the Material world geometric shapes can only approximate what exists as permanent and "uncreated" within Platonic abstraction. For example, a real orange is "inferior," temporal and irregular compared to an abstract, Platonic sphere.